$0^+(T=1)$ State and the Residual Interaction in Odd-Odd Deformed Nuclei*

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The $0^+(T=1)$ low-lying states in the nuclei F^{18}, Na^{22}, and Al^{26} show a regularity that is readily explained when each of these nuclei is described as an even-even core rotator with two odd nucleons coupled to it. It is shown that an acceptable short-range residual interaction between the two nucleons, together with the observed moments of inertia in this region, quantitatively reproduce that regularity.

I. INTRODUCTION

NUCLEI in the mass region $18 \leqslant A \leqslant 27$ have rotational properties, and are believed to be strongly deformed. The spherical j-j coupling orbits are mixed in a spheroidal field, and only k, the z component of single-particle angular momentum, is a good quantum number. It was shown[1] that a small number of nucleons outside the O^{16} core is sufficient to cause the large deformation, and at $A=18$ the nuclei are already strongly deformed. We are thus provided with a set of single-particle orbitals, each fourfold degenerate, which are filled in order: First a $k=\frac{1}{2}$ orbit, completed at Ne^{20}, then a $k=\frac{3}{2}$ at Mg^{24}, followed by a $k=\frac{5}{2}$ completed at Si^{28}, etc. The ground-state spins of odd-even nuclei in the first half of the s-d shell, are practically all correctly predicted by that picture.

II. THE ROTATIONAL MODEL FOR THE ODD-ODD NUCLEI

The odd-odd nuclei F^{18}, Na^{22}, Al^{26} have ground-state spins of 1^+, 3^+, 5^+, respectively, with $T=0$, indicating that the odd proton and odd neutron in the k orbit have their angular momenta aligned to produce a state of spin $I=2k$. In all these odd-odd nuclei there is a low-lying excited state with $I=0^+$; $T=1$, in which the two odd nucleons have their angular momenta antiparallel. The energy difference between the lowest $T=1$ state and the ground state may be computed using a simple rotational picture. The odd-odd nucleus is described as an even-even core, a rotator with parameter A with two odd nucleons coupled to it, the Hamiltonian is

$$H=A\mathbf{R}^2+h_1+h_2+V^{\mathrm{Res}}{}_{12}, \tag{1}$$

where $\mathbf{R}$ is the angular momentum of the core; h_1 and h_2 the single-particle deformed Hamiltonians; $V^{\mathrm{Res}}{}_{12}$ the residual two-body interaction between the two odd nucleons after a certain part of the total two-body interaction has been taken care of, by producing the deformed field. The angular momentum of the core is not a constant of the motion and is more conveniently written as

$$\mathbf{R}=\mathbf{I}-\mathbf{j}_1-\mathbf{j}_2, \tag{2}$$

where $\mathbf{I}$ is the total angular momentum. Therefore,

$$\begin{aligned} H&=A(\mathbf{I}-\mathbf{j}_1-\mathbf{j}_2)^2+h_1+h_2+V^{\mathrm{Res}}{}_{12}\\ &=A\mathbf{I}^2+A\mathbf{j}_1{}^2+A\mathbf{j}_2{}^2-2A\{(\mathbf{j}_1\cdot\mathbf{I})+(\mathbf{j}_2\cdot\mathbf{I})\}\\ &\qquad+2A(\mathbf{j}_1\cdot\mathbf{j}_2)+h_1+h_2+V^{\mathrm{Res}}{}_{12}. \end{aligned} \tag{3}$$

The ground state is described in an analogous way to the rotator particle treatment[2] as

$$\begin{aligned}\Psi^{\mathrm{G.S.}}\sim& D_{M\ 2k}{}^{2k}(\theta_i)\chi_k(1)\chi_k(2)\\ &+(-1)^{2k-j_1-j_2}D_{M\ -2k}{}^{2k}(\theta_i)\chi_{-k}(1)\chi_{-k}(2),\end{aligned} \tag{4}$$

where χ_k are eigenfunctions of h and θ_i are the angle variables in the laboratory fixed system. The $T=1$ excited state is described by

$$\Psi^{\mathrm{E.S.}}\sim\chi_k(1)\chi_{-k}(2)-\chi_{-k}(1)\chi_k(2). \tag{5}$$

The energy difference may be readily computed from

$$\Delta_k=\langle\Psi^{\mathrm{E.S.}}|H|\Psi^{\mathrm{E.S.}}\rangle-\langle\Psi^{\mathrm{G.S.}}|H|\Psi^{\mathrm{G.S.}}\rangle. \tag{6}$$

All the terms involving $\mathbf{I}$ vanish in the $I=0$ state; the $\mathbf{j}^2$ and the h terms contribute nothing to the difference, since

$$\langle\chi_k|\mathbf{j}^2|\chi_k\rangle=\langle\chi_{-k}|\mathbf{j}^2|\chi_{-k}\rangle$$

and

$$\langle\chi_k|h|\chi_k\rangle=\langle\chi_{-k}|h|\chi_{-k}\rangle.$$

We therefore have

$$\begin{aligned}\Delta_k=2A\langle\Psi^{\mathrm{E.S.}}|(\mathbf{j}_1\cdot\mathbf{j}_2)|\Psi^{\mathrm{E.S.}}\rangle-A\langle\Psi^{\mathrm{G.S.}}|\mathbf{I}^2-2(\mathbf{I}\cdot j_1)\\ -2(\mathbf{I}\cdot j_2)+2(j_1\cdot j_2)|\Psi^{\mathrm{G.S.}}\rangle+\delta_k,\end{aligned} \tag{7}$$

where

$$\delta_k=\langle\Psi^{\mathrm{E.S.}}|V^{\mathrm{Res}}{}_{12}|\Psi^{\mathrm{E.S.}}\rangle-\langle\Psi^{\mathrm{G.S.}}|V^{\mathrm{Res}}{}_{12}|\Psi^{\mathrm{G.S.}}\rangle. \tag{8}$$

Finally, evaluating explicitly, we obtain

$$\Delta_k=\delta_k-2Ak. \tag{9}$$

A decoupling Coriolis term correction $-a^2A$ is neglected in (9) in the $k=\frac{1}{2}$ case. (A detailed calculation shows the Coriolis term for this case to be of the order of 0.05 MeV.)

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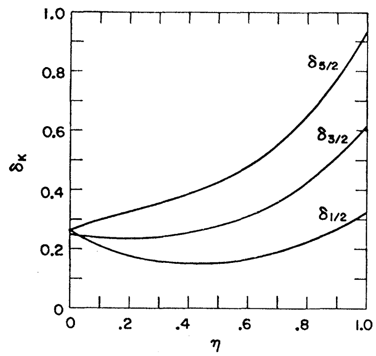

FIG. 1. $\delta_{1/2}$, $\delta_{3/2}$, and $\delta_{5/2}$ in MeV as a function of the dimensionless range parameter η. $V_0\eta^{3/2}$ is unity. $\eta=0$ corresponds to the short-range limit.

The collective parameter A for the nuclei in the first half of the s-d shell is fairly constant. The 0^+–4^+ energy differences in the rotational bands are:

4.31 MeV for O^{16} (excited state band),

4.25 MeV for Ne^{20} (ground state band),

4.12 MeV for Mg^{24} (ground state band).

(The 2^+ states are probably more strongly perturbed.) Thus, a value of $A\cong 0.2$, which is approximately obtained by various theoretical approaches as well,[3] should be inserted in (9).

The experimental Δ_k are as follows:

Nucleus	k	Δ_k (in MeV)
F^{18}	1/2	1.043
Na^{22}	3/2	0.660
Al^{26}	5/2	0.229

Δ_k behaves as an arithmetic series in k with decrement $2A\cong 0.4$ MeV. This is in complete accordance with the observed A and with (9) if we assume that δ_k is independent of k. It should be noted that the derivation of (9) involves only the general assumptions of the rotational picture.

III. THE RESIDUAL INTERACTION

It will now be shown that a typical residual interaction, $V_{TS}V(|r_{12}|)$, of a very short range gives δ_k, practically a constant δ, independent of k.

Since the exact radial dependence of the force hardly effects calculations, a Gaussian

$$V(|r_{12}|)=V_0\exp(-|r_{12}|/a)^2 \qquad (10)$$

is being chosen for convenience. The spin-isospin dependence is taken as the Rosenfeld mixture.[4] V_{TS} therefore has the following eigenvalues:

$$V_{00}=9/5 \quad V_{01}=-1 \quad V_{10}=-\tfrac{3}{5} \quad V_{11}=\tfrac{1}{3}. \qquad (11)$$

In the harmonic oscillator representation of range b, the interaction (10) is expandable[5] in Laguerre polynomials of order $\frac{1}{2}$; thus,

$$V(|r_{12}|)=\sum V_0\eta^{3/2}(1-\eta)^n L_n^{(1/2)}(\rho), \qquad (12)$$

where

$$\rho=|r_{12}|^2/2b^2 \qquad (13)$$

$$\eta=a^2/(a^2+2b^2). \qquad (14)$$

[For the matrix elements of $L_n^{(1/2)}(\rho)$ in the s-d shell, see, for example, Ref. 5.]

The states χ_k in Eqs. (4) and (5) should, in principle, be taken as eigenfunctions of a self-consistent single-particle Hamiltonian.[6] For the nuclei in question, the results of the self-consistent calculations[7] show that for the present purpose one could take

$$\chi_k=d_k^{5/2}. \qquad (15)$$

δ_k would then become

$$\delta_k=\langle d_k^{5/2}d_{-k}^{5/2}|V^{\mathrm{Res}}{}_{12}(T=1)|d_k^{5/2}d_{-k}^{5/2}\rangle - \langle d_k^{5/2}d_k^{5/2}|V^{\mathrm{Res}}{}_{12}(T=0)|d_k^{5/2}d_k^{5/2}\rangle. \qquad (16)$$

Figure 1 shows the dependence of $\delta_{1/2}$, $\delta_{3/2}$, and $\delta_{5/2}$ on the parameter η, the short-range limit corresponding to $\eta=0$. $V_0\eta^{3/2}$ is kept unity for all values of η in the graph. The figure shows that at the short-range limit δ_k is independent of k.

The fact that $V_0\eta^{3/2}$ is held constant means, in the short-range limit, that the strength×volume V_0a^3 is held constant. If we take in that limit the same value of $V_0\eta^{3/2}$, which is being successfully used for the finite-range two-body interaction in other works[6] ($V_0=40$ MeV; $\delta\simeq 5.2$ MeV), the constant $\delta\simeq 1.3$ MeV fits Eq. (9) in its absolute value as well. There is, however, no independent justification for this particular choice.

IV. DISCUSSION

The physical interpretation of those fits is still debatable. It could be argued that it is the long-range behavior of the two-body force that is responsible for the deformation of the nuclear shape and the occurrence of rotational phenomena. (This is indeed the reason why model forces of $P_2+\delta$ force have been extensively used.)[8] In that case, a short-range interaction between the two odd particles would mean that $A\mathbf{R}^2+h_1+h_2$ self-consistently represents the actual Hamiltonian for both the $T=0$ and $T=1$ states. Then the use of short-range force in the present calculation is justified.

Finally, we should add that the regularity of the $0^+(T=1)$ states in those odd-odd nuclei may have a significance independent of the particular assumptions of the model presented above.

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